1. $f(x) = 4 \operatorname{cosec} x - 4x + 1$, where x is in radians.

(a) Show that there is a root α of f $(x) = 0$ in the interval [1.2, 1.3].

(2)

(b) Show that the equation $f(x) = 0$ can be written in the form

$$x = \frac{1}{\sin x} + \frac{1}{4}$$

(2)

(c) Use the iterative formula

$$x_{n+1} = \frac{1}{\sin x_n} + \frac{1}{4}, \quad x_0 = 1.25,$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 4 decimal places.

(3)

(d) By considering the change of sign of $f(x)$ in a suitable interval, verify that $\alpha = 1.291$ correct to 3 decimal places.

(2)
(Total 9 marks)

2. $f(x) = x^3 + 2x^2 - 3x - 11$

(a) Show that $f(x) = 0$ can be rearranged as

$$x = \sqrt{\left(\frac{3x + 11}{x + 2}\right)}, \qquad x \neq -2.$$

The equation $f(x) = 0$ has one positive root α.

(2)

The iterative formula $x_{n+1} = \sqrt{\left(\frac{3x_n + 11}{x_n + 2}\right)}$ is used to find an approximation to α.

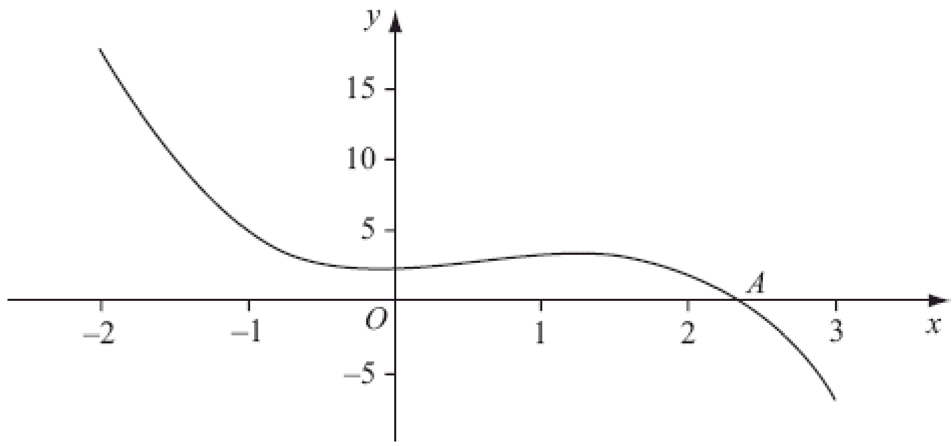
(b) Taking $x_1 = 0$, find, to 3 decimal places, the values of x_2, x_3 and x_4.

(3)

(c) Show that $\alpha = 2.057$ correct to 3 decimal places.

(3)
(Total 8 marks)

3.



The diagram above shows part of the curve with equation $y = -x^3 + 2x^2 + 2$, which intersects the x-axis at the point A where $x = \alpha$.

To find an approximation to α, the iterative formula

$$x_{n+1} = \frac{2}{(x_n)^2} + 2$$

is used.

(a) Taking $x_0 = 2.5$, find the values of x_1, x_2, x_3 and x_4.
Give your answers to 3 decimal places where appropriate.

(3)

(b) Show that $\alpha = 2.359$ correct to 3 decimal places.

(3)
(Total 6 marks)

4. $$f(x) = 3xe^x - 1$$

The curve with equation $y = f(x)$ has a turning point P.

(a) Find the exact coordinates of P.

(5)

The equation $f(x) = 0$ has a root between $x = 0.25$ and $x = 0.3$

(b) Use the iterative formula

$$x_{n+1} = \frac{1}{3}e^{-x_n}$$

with $x_0 = 0.25$ to find, to 4 decimal places, the values of x_1, x_2 and x_3.

(3)

(c) By choosing a suitable interval, show that a root of $f(x) = 0$ is $x = 0.2576$ correct to 4 decimal places.

(3)
(Total 11 marks)

5. $$f(x) = 3x^3 - 2x - 6$$

(a) Show that $f(x) = 0$ has a root, α, between $x = 1.4$ and $x = 1.45$

(2)

(b) Show that the equation $f(x) = 0$ can be written as

$$x = \sqrt{\left(\frac{2}{x} + \frac{2}{3}\right)}, \ x \neq 0.$$

(3)

(c) Starting with $x_0 = 1.43$, use the iteration

$$x_{n+1} = \sqrt{\left(\frac{2}{x_n} + \frac{2}{3}\right)}$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 4 decimal places.

(3)

(d) By choosing a suitable interval, show that $\alpha = 1.435$ is correct to 3 decimal places.

(3)
(Total 11 marks)

6. $$f(x) = \ln(x + 2) - x + 1,\ x > -2,\ x \in \mathbb{R}.$$

(a) Show that there is a root of $f(x) = 0$ in the interval $2 < x < 3$.

(2)

(b) Use the iterative formula

$$x_{n+1} = 1\text{n}(x_n + 2) + 1, \quad x_0 = 2.5$$

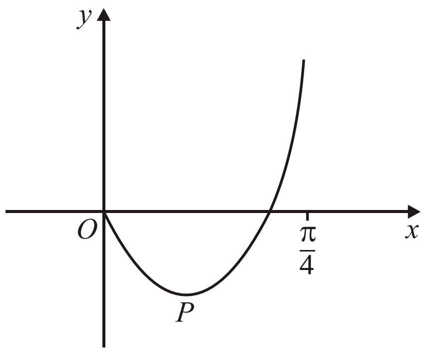
to calculate the values of x_1, x_2 and x_3 giving your answers to 5 decimal places.

(3)

(c) Show that $x = 2.505$ is a root of $f(x) = 0$ correct to 3 decimal places.

(2)
(Total 7 marks)

7.



The figure above shows part of the curve with equation

$$y = (2x - 1)\tan 2x, \quad 0 \le x < \frac{\pi}{4}$$

The curve has a minimum at the point P. The x-coordinate of P is k.

(a) Show that k satisfies the equation

$$4k + \sin 4k - 2 = 0.$$

(6)

The iterative formula

$$x_{n+1} = \frac{1}{4}(2 - \sin 4x_n), \qquad x_0 = 0.3,$$

is used to find an approximate value for k.

(b) Calculate the values of x_1, x_2, x_3 and x_4, giving your answers to 4 decimal places.

(3)

(c) Show that $k = 0.277$, correct to 3 significant figures.

(2)
(Total 11 marks)

8.

$$f(x) = 2x3 - x - 4.$$

(a) Show that the equation $f(x) = 0$ can be written as

$$x = \sqrt{\left(\frac{2}{x} + \frac{1}{2}\right)}$$

(3)

The equation $2x^3 - x - 4 = 0$ has a root between 1.35 and 1.4.

(b) Use the iteration formula

$$x_{n+1} = \sqrt{\left(\frac{2}{x} + \frac{1}{2}\right)},$$

with $x_0 = 1.35$, to find, to 2 decimal places, the values of x_1, x_2 and x_3.

(3)

The only real root of $f(x) = 0$ is α.

(c) By choosing a suitable interval, prove that $\alpha = 1.392$, to 3 decimal places.

(3)
(Total 9 marks)

9. $f(x) = 3e^x - \frac{1}{2}\ln x - 2, \quad x > 0.$

(a) Differentiate to find $f'(x)$.

(3)

The curve with equation $y = f(x)$ has a turning point at P. The x-coordinate of P is α.

(b) Show that $\alpha = \frac{1}{6}e^{-\alpha}$.

(2)

The iterative formula

$$x_{n+1} = \tfrac{1}{6}e^{-x_n}, \quad x_0 = 1,$$

is used to find an approximate value for α.

(c) Calculate the values of x_1, x_2, x_3 and x_4, giving your answers to 4 decimal places.

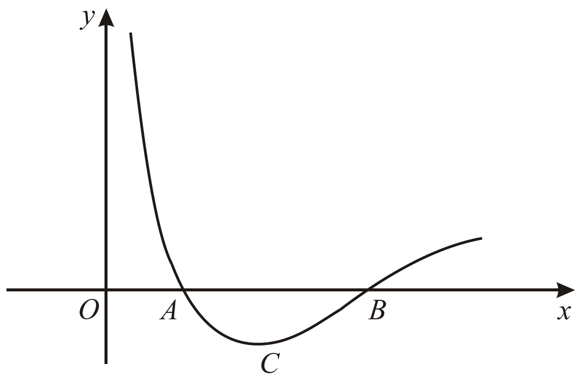
(2)

(d) By considering the change of sign of f ′(x) in a suitable interval, prove that $\alpha = 0.1443$ correct to 4 decimal places.

(2)

(Total 9 marks)

10.



$$f(x) = \frac{1}{2x} - 1 + \ln\frac{x}{2}, \; x > 0.$$

The diagram above shows part of the curve with equation $y = f(x)$. The curve crosses the x-axis at the points A and B, and has a minimum at the point C.

(a) Show that the x-coordinate of C is $\tfrac{1}{2}$.

(5)

(b) Find the y-coordinate of C in the form $k \ln 2$, where k is a constant.

(2)

(c) Verify that the x-coordinate of B lies between 4.905 and 4.915.

(2)

(d) Show that the equation $\frac{1}{2x} - 1 + \ln \frac{x}{2} = 0$ can be rearranged into the form $x = 2e^{\left(1 - \frac{1}{2x}\right)}$.

(2)

The x-coordinate of B is to be found using the iterative formula

$$x_{n+1} = 2e^{\left(1 - \frac{1}{2x_n}\right)}, \qquad \text{with } x_0 = 5.$$

(e) Calculate, to 4 decimal places, the values of x_1, x_2 and x_3.

(2)
(Total 13 marks)

11.

$$f(x) = x^3 - 2 - \frac{1}{x}, \quad x \neq 0.$$

(a) Show that the equation $f(x) = 0$ has a root between 1 and 2.

(2)

An approximation for this root is found using the iteration formula

$$x_{n+1} = \left(2 + \frac{1}{x_n}\right)^{\frac{1}{3}}, \quad \text{with } x_0 = 1.5.$$

(b) By calculating the values of x_1, x_2, x_3 and x_4, find an approximation to this root, giving your answer to 3 decimal places.

(4)

(c) By considering the change of sign of $f(x)$ in a suitable interval, verify that your answer to part (b) is correct to 3 decimal places.

(2)
(Total 8 marks)

12.

$$f(x) = x^3 + x^2 - 4x - 1.$$

The equation f(x) = 0 has only one positive root, α.

(a) Show that f(x) = 0 can be rearranged as

$$x = \sqrt{\left(\frac{4x+1}{x+1}\right)},\ x \neq -1.$$

(2)

The iterative formula $x_{n+1} = \sqrt{\left(\frac{4x_n+1}{x_n+1}\right)}$ is used to find an approximation to α.

(b) Taking $x_1 = 1$, find, to 2 decimal places, the values of x_2, x_3 and x_4.

(3)

(c) By choosing values of x in a suitable interval, prove that $\alpha = 1.70$, *correct* to 2 decimal places.

(3)

(d) Write down a value of x_1 for which the iteration formula $x_{n+1} = \sqrt{\left(\frac{4x_n+1}{x_n+1}\right)}$ does *not* produce a valid value for x_2.

Justify your answer.

(2)

(Total 10 marks)

13. (a) Sketch, on the same set of axes, the graphs of

$$y = 2 - e^{-x} \quad \text{and } y = \sqrt{x}.$$

[It is not necessary to find the coordinates of any points of intersection with the axes.]

(3)

Given that $f(x) = e^{-x} + \sqrt{x} - 2, \quad x \geq 0$,

(b) explain how your graphs show that the equation $f(x) = 0$ has only one solution,

(1)

(c) show that the solution of $f(x) = 0$ lies between $x = 3$ and $x = 4$.

(2)

The iterative formula $x_{n+1} = (2 - e^{-x_n})^2$ is used to solve the equation $f(x) = 0$.

(d) Taking $x_0 = 4$, write down the values of x_1, x_2, x_3 and x_4, and hence find an approximation to the solution of $f(x) = 0$, giving your answer to 3 decimal places.

(4)
(Total 10 marks)

14. The curve with equation $y = \ln 3x$ crosses the x-axis at the point P $(p, 0)$.

(a) Sketch the graph of $y = \ln 3x$, showing the exact value of p.

(2)

The normal to the curve at the point Q, with x-coordinate q, passes through the origin.

(b) Show that $x = q$ is a solution of the equation $x^2 + \ln 3x = 0$.

(4)

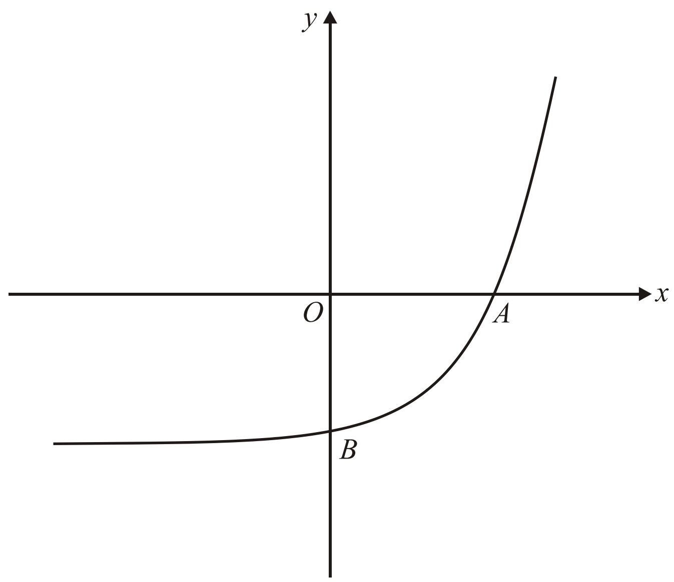
(c) Show that the equation in part (b) can be rearranged in the form $x = \frac{1}{3}e^{-x^2}$.

(2)

(d) Use the iteration formula $x_{n+1} = \frac{1}{3}e^{-x_n^2}$, with $x_0 = \frac{1}{3}$, to find x_1, x_2, x_3 and x_4. Hence write down, to 3 decimal places, an approximation for q.

(3)
(Total 11 marks)

15.



The diagram above shows a sketch of the curve with equation $y = \text{f}(x)$ where the function f is given by

$$\text{f}: x \mapsto \text{e}^{x-2} - 1, \quad x \in \mathbb{R}.$$

The curve meets the x-axis at the point A and the y-axis at the point B.

(a) Write down the coordinates of A and B.

(2)

(b) Find, in the form $\text{f}^{-1}(x): x \mapsto \ldots$, the inverse function of f and state its domain.

(5)

(c) Prove that the equation $\text{f}(x) = x$ has a root α in the interval [3, 4].

(2)

(d) Use the iterative formula

$$x_{n+1} = \text{f}^{-1}(x_n), \quad \text{with } x_1 = 3.5,$$

to find α to 3 decimal places. Prove that your answer is correct to 3 decimal places.

(5)
(Total 14 marks)

16. (a) Sketch the curve with equation $y = \ln x$.

(2)

(b) Show that the tangent to the curve with equation $y = \ln x$ at the point (e, 1) passes through the origin.

(3)

(c) Use your sketch to explain why the line $y = mx$ cuts the curve $y = \ln x$ between $x = 1$ and $x = \mathrm{e}$ if $0 < m < \frac{1}{\mathrm{e}}$.

(2)

Taking $x_0 = 1.86$ and using the iteration $x_{n+1} = \mathrm{e}^{\frac{1}{3}x_n}$,

(d) calculate x_1, x_2, x_3, x_4 and x_5, giving your answer to x_5 to 3 decimal places.

(3)

The root of $\ln x - \frac{1}{3}x = 0$ is α.

(e) By considering the change of sign of $\ln x - \frac{1}{3}x$ over a suitable interval, show that your answer for x_5 is an accurate estimate of α, correct to 3 decimal places.

(3)
(Total 13 marks)

1. (a) $f(1.2) = 0.49166551...,\quad f(1.3) = -0.048719817...$

Sign change (and as $f(x)$ is continuous) therefore a root α is such that $\alpha \in [1.2, 1.3]$ A1 2

Note

Attempts to evaluate both f(1.2) and f(1.3) and evaluates at least one of them correctly to awrt (or truncated) 1 sf.

A1: both values correct to awrt (or truncated) 1 sf, sign change and conclusion.

(b) $4\text{cosec}x - 4x + 1 = 0 \Rightarrow 4x = 4\text{cosec}x + 1$

$\Rightarrow x = \text{cosec}x + \frac{1}{4} \Rightarrow \underline{x = \frac{1}{\sin x} + \frac{1}{4}}$ A1 * 2

Note

Attempt to make 4x or x the subject of the equation.
A1: Candidate must then rearrange the equation to give the required result. It must be clear that candidate has made their initial f(x) = 0.

(c) $x_1 = \frac{1}{\sin(1.25)} + \frac{1}{4}$

$x_1 = 1.303757858...,\quad x_2 = 1.286745793...$ A1

$x_3 = 1.291744613...$ A1 3

Note

An attempt to substitute x_0 = 1.25 into the iterative formula

Eg = $\frac{1}{\sin(1.25)} + \frac{1}{4}$

Can be implied by x_1 = awrt 1.3 or x_1 = awrt 46°.

A1: Both x_1 = awrt 1.3038 and x_2 = awrt 1.2867

A1: x_3 = awrt 1.2917

(d) $f(1.2905) = 0.00044566695...,\quad f(1.2915) = -0.00475017278...$

Sign change (and as $f(x)$ is continuous) therefore a root α is such that

$\alpha \in (1.2905, 1.2915) \Rightarrow \alpha = 1.291$ (3 dp) A1 2

Note

Choose suitable interval for x, e.g. [1.2905, 1.2915] or tighter and at least one attempt to evaluate f(x).
A1: both values correct to awrt (or truncated) 1 sf, sign change and conclusion.

[9]

2. (a) $f(x) = x^3 + 2x^2 - 3x - 11$

Sets $f(x) = 0$ (can be implied)

$f(x) = 0 \;\Rightarrow\; x^3 + 2x^2 - 3x - 11 = 0$ — and takes out a factor

$\Rightarrow\; x^2(x + 2) - 3x - 11 = 0$ — of x^2 from $x^3 + 2x^2$,

or x from $x^3 + 2x$ (slip).

$\Rightarrow\; x^2(x + 2) - 3x - 11$

$\Rightarrow\; x^2 = \dfrac{3x+11}{x+2}$

$\Rightarrow\; x = \sqrt{\left(\dfrac{3x+11}{x+2}\right)}$ — then rearranges to give

the quoted result on the question paper. A1 **AG** 2

(b) Iterative formula:

$x_{n+1} = \sqrt{\left(\dfrac{3x_n+11}{x_n+2}\right)},\ x_1 = 0$

$x_2 = \sqrt{\left(\dfrac{3(0)+11}{(0)+2}\right)}$ — An attempt to substitute $x_1 = 0$

into the iterative formula.

Can be implied by $x_2 = \sqrt{5.5}$

or 2.35 or awrt 2.345

$x_2 = 2.34520788...$ — Both x_2 = awrt 2.345 A1

$x_3 = 2.037324945...$ — and x_3 = awrt 2.037

$x_4 = 2.058748112...$ — x_4 = awrt 2.059 A1 3

(c) Let $f(x) = x^3 + 2x^2 - 3x - 11 = 0$

f (2.0565) = – 0.013781637...

f (2.0575) = 0.0041401094...

Sign change (and f(x) is continuous) therefore
a root α is such that $\alpha \in (2.0565, 2.0575)$
$\Rightarrow \alpha = 2.057$ (3 dp)

Choose suitable interval for x, e.g. [2.0565, 2.0575] or tighter

any one value awrt 1 sf — dM1

both values correct awrt 1sf, sign change and conclusion(*) — A1 — 3

(*) As a minimum, both values must be correct to 1sf, candidate states "change of sign, hence root"

[8]

3. (a) Iterative formula: $x_{n+1} = \frac{2}{(x_n)^2} + 2, \; x_0 = 2.5$

$x_1 = \frac{2}{(2.5)^2} + 2$ — An attempt to substitute $x_0 = 2.5$ into the iterative formula. Can be implied by $x_1 = 2.32$ or 2.320

$x_1 = 2.32$
$x_2 = 2.371581451...$ — Both $x_1 = 2.32(0)$ and x_2 = awrt 2.372 — A1
$x_3 = 2.355593575...$
$x_4 = 2.360436923...$ — Both x_3 = awrt 2.356 and x_4 = awrt 2.360 or 2.36 — A1 cso — 3

(b) Let $f(x) = -x^3 + 2x^2 + 2 = 0$

f (2.3585) = 0.00583577...
f (2.3595) = – 0.00142286...
Sign change (and f (x) is continuous) therefore a root
α is such that $\alpha \in (2.3585, 2.3595) \Rightarrow \alpha = 2.359$ (3 dp)

Choose suitable interval for x, e.g. [2.3585, 2.3595] or tighter
any one value awrt 1 sf or truncated 1 sf — dM1
both values correct, sign change and conclusion — A1 — 3
At a minimum, both values must be
At a minimum, both values must be correct to 1sf or truncated 1sf, candidate states "change of sign, hence root".

[6]

4.	(a)	$f'(x) = 3e^x + 3xe^x$	A1	
		$3e^x + 3xe^x = 3ex\ (1+x) = 0$		
		$x = -1$	M1A1	
		$f(-1) = -3e^{-1} - 1$	B1	5
	(b)	$x_1 = 0.2596$	B1	
		$x_2 = 0.2571$	B1	
		$x_3 = 0.2578$	B1	3
	(c)	Choosing (0.257 55, 0.257 65) or an appropriate tighter interval.		
		f(0.257 55) = – 0.000 379 ...		
		f(0.257 65)= 0.000109 …	A1	
		Change of sign (and continuity) $\Rightarrow$ root $\in (0.257\,55, 0.257\,65)$ * cso	A1	3
		($\Rightarrow$ $x = 0.2576$, is correct to 4 decimal places)		
		Note:		
		x = 0.257 627 65 … is accurate		

[11]

5.	(a)	f(1.4) = –0.568 … < 0			
		f(1.45) = 0.245 … > 0			
		Change of sign (and continuity) $\Rightarrow \alpha$ (1.4, 1.45)		A1	2
	(b)	$3x^3 = 2x + 6$			
		$x^3 = \frac{2x}{3} + 2$			
		$x^2 = \frac{2}{3} + \frac{2}{x}$		M1A1	
		$x = \sqrt{\left(\frac{2}{x} + \frac{2}{3}\right)}$ *	cso	A1	3
	(c)	$x_1 = 1.4371$		B1	
		$x_2 = 1.4347$		B1	
		$x_3 = 1.4355$		B1	3

(d) Choosing the interval (1.4345,1.4355) or appropriate tighter interval.
f (1.4345) = –0.01 ...
f(1.4355) = 0.003 …
Change of sign (and continuity) $\Rightarrow \alpha \in (1.4345, 1.4355)$
$\Rightarrow \alpha = 1.435$, correct to 3 decimal places * cso — A1 — 3

Note: $\alpha = 1.435\ 304\ 553 \ldots$

[11]

6. (a) f(2) = 0.38 …
f(3) = –0.39 …
Change of sign (and continuity) ⇒ root in (2, 3) (*) — cso — A1 — 2

(b) $x_1 = \ln 4.5 + 1 \approx 2.50408$
$x_2 \approx 2.50498$ — A1
$x_3 \approx 2.50518$ — A1 — 3

(c) Selecting [2.5045, 2.5055], or appropriate tighter range, and evaluating at both ends.
$f(2.5045) \approx 6 \times 10^{-4}$
$f(2.5055) \approx -2 \times 10^{-4}$
Change of sign (and continuity) ⇒ root ∈ (2.5045, 2.5055)
⇒ root = 2.505 to 3 dp (*) — cso — A1 — 2

Note: The root, correct to 5 dp, is 2.50524

[7]

7. (a) Using product rule: $\frac{dy}{dx} = 2\tan 2x + 2(2x-1)\sec^2 2x$ — A1 A1

Use of "$\tan 2x = \frac{\sin 2x}{\cos 2x}$" and "$\sec 2x = \frac{1}{\cos 2x}$"

$[= 2\frac{\sin 2x}{\cos 2x} + 2(2x-1)\frac{1}{\cos^2 2x}]$

Setting $\frac{dy}{dx} = 0$ and multiplying through to eliminate fractions
$[\Rightarrow 2\sin 2x \cos 2x + 2(2x - 1) = 0]$

Completion: producing $4k + \sin 4k - 2 = 0$ with no wrong working seen and at least previous line seen. — AG — A1 — 6

(b) $x_1 = 0.2670$, $x_2 = 0.2809$, $x_3 = 0.2746$, $x_4 = 0.2774$, A1 A1 3

for first correct application, first A1 for two correct, second A1 for all four correct

Max –1 deduction, if ALL correct to > 4 d.p. A0 A1

SC: degree mode: $x_1 = 0.4948$, A1 for $x_2 = 0.4914$, then A0; max 2

(c) Choose suitable interval for *k:* e.g. [0.2765, 0.2775]
and evaluate f(x) at these values

Show that $4k + \sin 4k - 2$ changes sign and deduction Al 2

[f(0.2765) = –0.000087.., f(0.2775) = +0.0057]

Continued iteration: (no marks in degree mode)

Some evidence of further iterations leading to 0.2765 or better

Deduction A1

[11]

8. (a) $x(2x^2 - 1) = 4$

$$2x^2 - 1 = \frac{4}{x}$$

$$2x^2 = \frac{4+x}{x}$$

$$x^2 = \frac{4+x}{2x}$$

$$x = \sqrt{\frac{2}{x} + \frac{1}{2}}$$ **AG** A1 3

Alternative 1:

$$2x^2 - 1 - \frac{4}{x} = 0$$

$$2x^2 = 1 + \frac{4}{x}$$

$$x^2 = \frac{1}{2} + \frac{4}{2x}$$

$$x\sqrt{\frac{1}{2} + \frac{2}{x}}$$ **AG** A1

Alternative 2:

$x^2 = \frac{2}{x} + \frac{1}{2}$

$2x^3 = 4 + x$

$2x^2 - x - 4 = 0$ A1

(b) 1.41, 1.39, 1.39 B1,B1,B1 3
(1.40765, 1.38593, 1.393941)

(c) $f(1.3915) = -3 \times 10^{-3}$
$f(1.3925) = 7 \times 10^{-3}$ A1

change in sign means root between
1.3915 & 1.3925
$\therefore$ 1.392 to 3 dp B1 3

[9]

9. (a) $f'(x) = 3e^x - \frac{1}{2x}$ M1A1A1 3

any evidence to suggest that tried to differentiate

(b) $3e^{\alpha} - \frac{1}{2\alpha} = 0$

Equating f′(x) to zero

$\Rightarrow 6\alpha e^{\alpha} = 1 \Rightarrow \alpha = \frac{1}{6}e^{-\alpha}$ AG A1(cso) 2

(c) $x_1 = 0.0613\ldots$, $x_2 = 0.1568\ldots$, $x_3 = 0.1425\ldots$, $x_4 = 0.1445\ldots$ M1A1 2

at least x_1 correct, A1 all correct to 4 d.p.

(d) Using $f'(x)$ $\{= 3e^x - \frac{1}{2x}\}$ with suitable interval
[e.g. f(0.14425) = – 0.007, f(0.14435) = +0.002(1)]
Both correct with concluding statement. A1 2

[9]

10. (a) $f'(x) = -\frac{1}{2x^2};+\frac{1}{x}$ M1A1;A1

for evidence of differentiation. Final A – no extras

$f'(x) = 0 \Rightarrow \frac{-1+2x}{2x^2} = 0; \Rightarrow x = 0.5$ M1A1 (*) cso 5

(or subst x = 0.5)

(b) $y = 1 - 1 + \ln\left(\frac{1}{4}\right); = -2\ln 2$ A1 2

Sust 0.5 or their value for x in

(c) f(4.905) = < 0 (–0.000955), f(4.915) = > 0 (+ 0.000874)

evaluate

Change of sign indicates root between and correct values to 1 sf) A1 2

(d) $\frac{1}{2x} - 1 + \ln\left(\frac{x}{2}\right) = 0; \quad \Rightarrow 1 - \frac{1}{2x} = \ln\left(\frac{x}{2}\right)$

$\Rightarrow \frac{x}{2} = e^{\left(1-\frac{1}{2x}\right)}; \Rightarrow x = 2e^{\left(1-\frac{1}{2x}\right)}$ (*) (c.s.o.) A1 2

for use of e to the power on both sides

(e) $x_1 = 4.9192$ B1

$x_2 = 4.9111$, $x_3 = 4.9103$, B1 2

both, only lose one if not 4dp

[13]

11. (a) f(1) = –2, f(2) = 5 ½

Change of sign (and continuity) $\Rightarrow$ root $\in (1, 2)$ A1 2

(b) $x_1 = 1.38672\ldots$, $x_2 = 1.39609$ awrt 4dp B1, B1

$x_3 = 1.39527\ldots$, $x_4 = 1.39534\ldots$ same to 3dp

Root is 1.395 (to 3dp) cao A1 4

(c) Choosing a suitable interval, (1.3945, 1.3955) or tighter.
f(1.3945) ≈ – 0.005, f(1.3955) ≈ + 0.001
Change of sign (and continuity) ⇒ root ∈ (1.3945, 1.3955)
⇒ root is 1.395 correct to 3dp A1 2

[8]

12. (a) Attempting to reach at least the stage $x^2(x + 1) = 4x + 1$

Conclusion (no errors seen) $x = \sqrt{\dfrac{4x+1}{x+1}}$ (*) A1 2

[Reverse process: need to square and clear fractions for

(b) $x_2 = \sqrt{\dfrac{4+1}{1+1}} =$ **1.58**…

$\mathbf{x_3 = 1.68}$**,** $\mathbf{x_4 = 1.70}$ A1A1 3

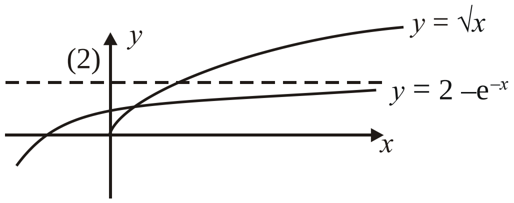
[Max. deduction of 1 for more than 2 d.p.]

(c) Suitable interval; e.g. [1.695, 1.705] (or "tighter")
f(1.695) = –0.037…, f(1.705) = + 0.0435… Dep.
Change of sign, no errors seen, so root = 1.70 (correct to 2 d.p.) A1 3

(d) $x = -1$, "division by zero not possible", or equivalent B1,B1 2
or **any number in interval –1 < *x* < –¼,** "square root of neg. no."

[10]

13. (a)



[Ignore graph for y < 0, x < 0]

$y = \sqrt{x}$: starting (0,0) B1

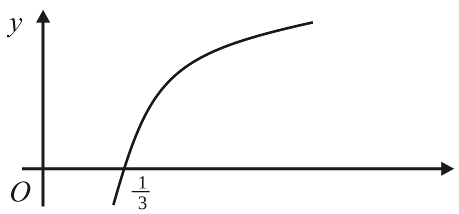
$y = 2 - e^{-x}$:
shape & int. on + y-axis B1

correct relative positions B1 3

[1 int^n √x on top for x → ∞]

(b)	Where curves meet is solution to f(x) = 0; <u>only one intersection</u>	B1	1	
(c)	f(3) = –0.218… f(4) = 0.018… *one correct value to 1 sf*			
	change of sign ∴ root in interval *both correct (1 sf) + comment*		2	
(d)	$x_0 = 4$ $x_1 = (2 - e^{-4})^2$ = 3.92707… *expression or x_1 to 3 dp*			
	x_2 = 3.92158… *x_1, x_2 to ≥ 4 dp*	A1		
	x_3 = 3.92115… *carry on to x_4*			
	x_4 = 3.92111(9)… *to ≥ 3 dp*			
	Approx. solution = <u>3.921</u> (3 dp)	A1 cao	4	
				[10]

14. (a)



	Shape	B1	
	$p = \frac{1}{3}$ or $\{\frac{1}{3}, 0\}$ seen	B1	2
(b)	Gradient of tangent at $Q = \dfrac{1}{q}$	B1	
	Gradient of normal = $-q$		
	Attempt at equation of OQ [$y = -qx$] and substituting $x = q$, $y = \ln 3q$		
	or attempt at equation of tangent [$y - 3 \ln q = -q(x - q)$] with $x = 0$, $y = 0$		
	or equating gradient of normal to $(\ln 3q)/q$		
	$q^2 + \ln 3q = 0$ (*)	A1	4

(c) $\ln 3x = -x^2 \Rightarrow 3x = e^{-x^2}$; $\Rightarrow x = \frac{1}{3}e^{-x^2}$ A1 2

(d) $x_1 = 0.298280$; $x_2 = 0.304957$, $x_3 = 0.303731$, $x_4 = 0.303958$ A1

Root = 0.304 (3 decimal places) A1 3

[11]

15. (a) A is (2, 0); B is (0, $e^{-2} - 1$) B1; B1 2

(b) $y = e^{x-2} - 1$

Change over x and y, $x = e^{y-2} - 1$

$y - 2 = \ln(x + 1)$

$y = 2 + \ln(x + 1)$ A1

$f^{-1} : x / 2 + \ln(x + 1), x > -1$ A1 A1 5

(c) $f(x) - x = 0$ is equivalent to $e^{x-2} - 1 - x = 0$

Let $g(x) = e^{x-2} - 1 - x$

$g(3) = -1.28\ldots$

$g(4) = 2.38\ldots$

Sign change $\Rightarrow$ root α A1 2

(d) $x_{n+1} = 2 + \ln(x_n + 1)$, $x_1 = 3.5$

$x_2 = 3.5040774$ A1

$x_3 = 3.5049831$ A1

$x_4 = 3.5051841$

$x_5 = 3.5052288$

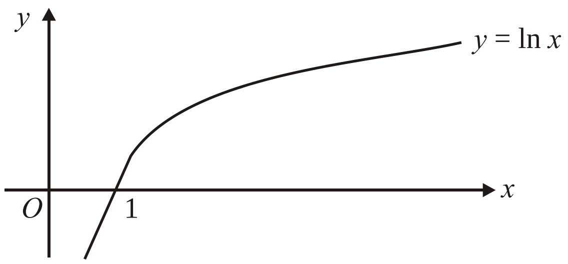
Needs convincing argument on 3 d.p. accuracy

Take 3.5053 and next iteration is reducing 3.50525…

Answer: 3.505 (3 d.p.) A1 5

[14]

16. (a)



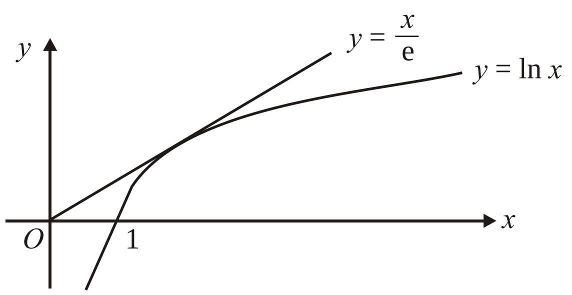
B1 shape

B1 x-intercept labelled 2

(b) $\frac{dy}{dx} = \frac{1}{x}$ so tangent line to (e, 1) is $y = \frac{1}{e}x + C$

the line passes through (e, 1) so $1 = e\frac{1}{e} + C$ and $C = 0$

The line passes through the origin. A1 3



(c) All lines $y = mx$ passing through the origin and having a gradient > 0 lie above the x-axis.

Those having a gradient < $\frac{1}{e}$ will lie below the line. B1

$y = \frac{x}{e}$ so it cuts $y = \ln x$ between $x = 1$ and $x = e$. B1 2

(d) $x_0 = 1.86$

$x_1 = e^{\frac{x_n}{3}} = 1.859$

$x_2 = 1.858$ A1

$x_3 = 1.858$

$x_4 = 1.858$

$x_5 = 1.857$ A1 3

(e)	When $x = 1.8575$, $\ln x - \frac{1}{3}x = 0.000\,064\,8\ldots > 0$		
	When $x = 1.8565$, $\ln x = -0.000\,140\ldots < 0$	A1	
	Change of sign implies there is a root between.	A1	3

[13]

1. All four parts of this question were well answered by the overwhelming majority of candidates who demonstrated their confidence with the topic of iteration with around 65% of candidates gaining at least 8 of the 9 marks available.

Some candidates in parts (a), (c) and (d) worked in degrees even though it was stated in the question that x was measured in radians.

In part (a), the majority of candidates evaluated both f(1.2) and f(1.3), although a very small number choose instead to evaluate both f(1.15)and f(1.35) A few candidates failed to conclude "sign change, hence root" as minimal evidence for the accuracy mark.

Most candidates found the proof relatively straightforward in part (b). A small number of candidates lost the accuracy mark by failing to explicitly write $4\cos cx - 4x + 1$ as equal to 0 as part of their proof.

Part (c) was almost universally answered correctly, although a few candidates incorrectly gave x_1 as 1.3037 or x_3 as 1.2918.

The majority of candidates who attempted part (d) choose an appropriate interval for x and evaluated f(x) at both ends of that interval. The majority of these candidates chose the interval (1.2905, 1.2915) although incorrect intervals, such as (1.290, 1.292)were seen. There were a few candidates who chose the interval (1.2905, 1.2914). This probably reflects a misunderstanding of the nature of rounding but a change of sign over this interval does establish the correct result and this was accepted for full marks. To gain the final mark, candidates are expected to give a reason that there is a sign change, and give a suitable conclusion such as that the root is 1.291 to 3 decimal places or $\alpha = 1.291$ or even QED.

A minority of candidates who attempted part (d) by using a repeated iteration technique received no credit because the question required the candidate to consider a change of sign of f(x).

2. All three parts of this question were well answered by the overwhelming majority of candidates who demonstrated their confidence with the topic of iteration.

Part (a) was well answered by the majority of candidates although a significant minority of candidates were not rigorous enough in their proof. Some candidates assumed the step of factorising a common factor of x^2 from their first two terms rather than explicitly showing it. A few candidates attempted to reverse the proof and arrived at the correct equation but many of these candidates lost the final accuracy mark by not referring to f (x) = 0.

Part (b) was almost universally answered correctly, although a few candidates incorrectly gave x_4 as 2.058.

The majority of candidates who attempted part (c) choose an appropriate interval for x and evaluated f (x) at both ends of that interval. The majority of these candidates chose the interval (2.0565, 2.0575) although incorrect intervals, such as (2.056, 2.058) were seen.

There were a few candidates who chose the interval (2.0565, 2.0574) . This probably reflects a misunderstanding of the nature of rounding but a change of sign over this interval does establish the correct result and this was accepted for full marks. To gain the final mark, candidates are expected to give a reason that there is a sign change, and give a suitable conclusion such as that the root is 2.057 to 3 decimal places or $\alpha = 2.057$ or even QED.

A minority of candidates attempted part (c) by using a repeated iteration technique. Almost all of these candidates iterated as far as x_6 (or beyond) but most of these did not write downtheir answers to at least four decimal places. Of those candidates who did, very few of themmanaged to give a valid conclusion.

3. This question was generally well answered with many successful attempts seen in both parts. There were few very poor or non-attempts at this question.

In part (a), the majority of candidates were able to score all three marks. There were a significant number of candidates in this part who incorrectly gave x_3 and x_4 as 2.355 and 2.361 respectively. These incorrect answers were usually achieved by candidates substituting the rounded answer of $x_2 = 2.372$ to find x_3 and substituting their rounded answer of $x_3 = 2.355$ to find x_4. Some candidates are not aware that it is possible to program a basic calculator by using the ANS button to find or even check all four answers. Another common error in this part was for candidates to stop after evaluating x_3.

The majority of candidates who attempted part (b) choose an appropriate interval for x and evaluated y at both ends of that interval. The majority of these candidates chose the interval (2.3585, 2.3595) although incorrect intervals, such as (2.358, 2.360) were seen. There were a few candidates who chose the interval (2.3585, 2.3594). This probably reflects a misunderstanding of the nature of rounding but a change of sign over this interval does establish the correct result and this was accepted for full marks. To gain the final mark, candidates are expected to give a reason that there is a sign change, and give a suitable conclusion such as that the root is 2.359 to 3 decimal places or $\alpha = 2.359$ or even QED.

A minority of candidates attempted part (b) by using a repeated iteration technique. Almost all of these candidates iterated as far as x_6 (or beyond) but most of these did not write down their answers to at least four decimal places. Of those candidates who did, very few candidates managed to give a valid conclusion.

4. A substantial proportion of candidates did not recognise that, in part (a), the product rule is needed to differentiate $3x\ e^x$ and $3x^2\ e^x$, $3x\ e^x$ and $3e^x$ were all commonly seen. It was also not uncommon for the question to be misinterpreted and for $3x\ (e^x - 1)$ to be differentiated. Those who did differentiate correctly usually completed part (a) correctly. Part (b) was very well done with the majority of the candidates gaining full marks. Very few lost marks for truncating their decimals or giving too many decimal places.

In parts (c), candidates need to be aware that showing that something is true requires them to give reasons and conclusions. It would be sufficient to argue that a change of sign in the interval(0.257 55, 0.257 65)implies that there is a root in the interval (0.257 55, 0.257 65) and, hence, that $x = 0.2576$ is correct to 4 decimal places. The majority of candidates did provide an acceptable argument. Fewer candidates than usual attempted repeated iteration, an method that is explicitly ruled out by the wording of the question.

5. In parts (a) and (d), candidates need to be aware that showing that something is true requires them to give reasons and conclusions. In this part (a), it is sufficient to say that a change of sign in the interval (1.4, 1.45) implies that there is a root in the interval (1.4, 1.45). In part (c), it would be sufficient to argue that a change of sign in the interval (1.4345, 1.4355) implies that there is a root in the interval (1.4345, 1.4355) and, hence, that $x = 1.435$ is accurate to 3 decimal places. Part (b) was very well done but candidates must put all steps in a proof and not leave it to the examiners to fill in important lines. Part (c) was very well done. Some candidates attempted part (d) using repeated iteration but the wording of the question precludes such a method and no marks could be gained this way.

6. In parts (a) and (c), candidates need to be aware that showing that something is true requires them to give reasons and conclusions. In this part (a), it is sufficient to say that a change of sign in the interval (2, 3) implies that there is a root in the interval (2, 3). In part (c), it would be sufficient to argue that a change of sign in the interval (2.5045, 2.5055) implies that there is a root in the interval (2.5045, 2.5055) and, hence, that $x =2.505$ is accurate to 3 decimal places.

In part (a), most candidates chose the obvious 2 and 3 and successfully found f(2) and f(3) to gain the method mark.

The majority of candidates now seem comfortable with the method of iteration. Part (b) was particularly well answered with only a minority of candidates making errors, mainly over issues of accuracy.

In part (c) candidates who chose (2.5045, 2.5055) were more often successful than not. Although it is not a wholly satisfactory method, on this occasion the examiners did accept repeated iteration. The candidates were required to reach at least x_6, showing their working to 5 decimal places, which most choosing this method did, and to give a reason why they concluded that the root was accurate to 3 decimal places. This second requirement was rarely met.

7. The product rule is well known and was accurately applied by many candidates in part (a). Rather than changing tan $2x$ to $\frac{\sin 2x}{\cos 2x}$ and $\sec^2 2x$ to $\frac{1}{\cos^2 2x}$ some candidates used the identity $1 + \tan^2 2x = \sec 2x$ These candidates were rarely able to make progress beyond a few more lines of manipulation and such solutions were often abandoned. Algebraic manipulation was a problem for some candidates. Others never set $\frac{dy}{dx}$ equal to zero and incorrectly multiplied only one side of their equation by $\cos^2 2x$ rather than using a common denominator or stating that $\frac{dy}{dx} = 0$ before multiplying by $\cos^2 2x$ This part of the question asked candidates to show a given result and candidates did not always show sufficient steps in their work. Full marks were not awarded unless the 4 sin k part of the equation came from an intermediate result of 2 sin 2 k 2 cos k somewhere in the solution. Many correct solutions were seen in (b), although a few candidates were inaccurate when giving their answers to 4 decimal places. By far the most common error came from candidates using their calculators in degree mode rather than radian mode. Part (c) was generally done well. Some candidates chose an unsuitably large interval and some worked in degrees. Candidates who performed further iterations gained the marks provided they showed sufficient accuracy in their answers.

8. **Pure Mathematics P2**

A few candidates made no attempt to link the original function with the iteration formula. For those who did attempt this part, the most popular route was to start with f(x) = 0 and attempt to rearrange it. There were a few algebraic slips, and some candidates who could not see how to achieve an x^2 term, but most were successful. The few candidates who worked in the reverse direction, working from the iteration formula towards f(x) were all successful.

Many candidates were successful in applying the iteration formula, and usually noticed the instruction to give x_1, x_2 and x_3 to 2 decimal places. A few candidates appeared to think that this instruction applied only to x_3. There were some candidates having difficulty in the correct use of their calculators; a common set of false answers resulted from the alternative formula $\sqrt{\frac{2}{x}} + \frac{1}{2}$, despite the original formula being quoted correctly.

The wording of the final part of this question clearly indicates the need for a method involving an interval, yet many candidates simply continued for several more applications of the iteration formula. This approach gained no credit. Some candidates continue to have difficulty in identifying the correct values to use for the endpoints of their interval. (1.391, 1.393) was quite a common error, as was candidates attempting to add and subtract 5 in the wrong decimal place. There were a good number of totally correct answers.

Core Mathematics

This question was well answered and many candidates gained full marks. The great majority were able to provide the appropriate sequence of steps to demonstrate the result in part (a) and part (b) was nearly always completely correct with the answers given to the accuracy requested. The method tested in part(c) was clearly known to more candidates than had been the case in some previous examinations but there is still a minority of candidates who think that repeating the iteration gives a satisfactory proof. Here the question specified that an interval should be used and further iterations could gain no marks. The majority chose the appropriate interval (1.3915, 1.3925) although incorrect intervals, such as (1.391, 1.393) were seen. There were candidates who chose the interval (1.3915, 1,3924). This probably reflects a misunderstanding of the nature of rounding but a change of sign over this interval does establish the result and this was accepted for full marks. To gain the last mark, candidates are expected to give a reason for their conclusion, e.g. there is a sign change, and give a suitable conclusion such as that the root is 1.392 to 3 decimal places, the traditional, QED or, the modern, W.

9. Pure Mathematics P2

Most candidates were able to complete some, if not all parts of this question.

(a) Most candidates scored well, with an inability to differentiate log x being the most common cause of error. This part of the question was subject to a common misread of the function as $f(x) = 3e^x - \frac{1}{2}\ln(x-2)$. We often see candidates working without correct used of brackets, and here they saw brackets that were not present.

(b) have been straight forward but was made difficult by several candidates. Many candidates managed to produce the correct answer to (b) from a completely wrong answer to (a)!

(c) This was well done - most candidates obviously know how to use the ANS button on their calculators. Many candidates who went wrong had either made rounding errors, or they had assumed that $e^{-1} = 1$ when making their first substitution.

(d) This part was found more difficult. Candidates who did use an appropriate interval did not always give the values of the derivative correctly (for example, we assumed that an answer of 2.06… was a misread of a value in standard form). Some candidates did all the working correctly but did not draw any conclusion from it. Many chose an interval that did not include the root or that was too wide. A significant number ignored the wording of the question and continued the iteration process.

Core Mathematics

(a) Most candidates were confident differentiating both e^x and ln x. Some candidates misread f(x) and had ln(x – 2) instead of lnx -2. A number did not simplify their fraction to 1/(2x) but they were not penalised for this.

(b) They usually completed the rearrangement in part b) successfully although often needing several stages before reaching the answer on the paper. Some failed to replace the x with an alpha and a number had problems with their algebraic fractions.

(c) Candidates are very competent at obtaining values using an iteration formulae but some are not precise about the required number of decimal places.

(d) There was general familiarity with the change of sign method for determining a root but not always sufficient decimal places, and some intervals were too wide. Conclusions were sometimes missing. Some answers used f instead of f', and others used an iterative approach despite the instruction in the question.

10. In part (a) many candidates wrote $1/2x$ as $2x^{-1}$ and differentiated to get $2x^{-2}$. Many candidates then converted $-2x^{-2}$ to $-1/2x^{2}$. Much fudging then went on to arrive at $x = ½$, particularly since a large number had *d(ln x/2)/dx = 2/x*. A fair number tried to show f(1/2) = 0. Part (b) Most candidates substituted in x = ½ correctly but a few did not know what to do with $\ln\frac{1}{4}$.

Most candidates knew what to do in part (c), though a few candidates did not actually evaluate f(4.905) or f(4.915) or did so incorrectly. The phrase "change of sign" was often not mentioned, but replaced with long convoluted statements. Part (d) was well answered but some candidates did miss lines out going from $\ln\frac{x}{2} = 1 - \frac{1}{2x}$ to $x = 2e^{1-\frac{1}{2x}}$

In part (e) the majority of candidates did well with some students though some lost marks because they could not use their calculator correctly or did not give their answers to the required number of decimal places.

11. Part (a) was well answered but in part (b) many did not work to an appropriate accuracy. In order to give a final answer to 3 decimal places, it is necessary to produce intermediate working to 4 decimal places. Many produced an incorrect iteration which arose from the incorrect use calculators. Calculators differ, but most, when given an instruction (2 + 1/*x*)^1/3, calculate $\frac{\left(2+\frac{1}{x}\right)^1}{3}$ not $\left(2+\frac{1}{x}\right)^{\frac{1}{3}}$. For the second of these expressions, the instruction (2 + 1/*x*)^(1/3) is needed. A correct solution to part (c) requires the use of the interval (1.3945, 1.3955) or of an appropriate smaller interval. Less than 40% of the candidates recognised this.

12. Most candidates were able to gain marks, particularly in parts (a) and (b), although some lost accuracy marks. In part (c) many candidates did not read, or interpret, the question carefully enough, and the most common offerings were just to produce another three iterations, or to look for a sign change in an interval which was too wide. Only the better candidates gained full marks here. Answers to the last part were variable. Often a correct value with a wrong reason was seen, and "$x_1 = -¼$, because $x_2 = 0$, which is not valid" was quite common.

13. The sketches in part (a) proved quite demanding for some candidates. The sketch of $y = \sqrt{x}$ usually passed through the origin but the curvature was often incorrect and a sketch that resembled $y = e^{-x}$ was often given for $y = 2 - e^{-x}$. Part (b) was usually answered correctly but a small number of candidates thought that this question was more to do with the number of intersections with the axes rather than between the two curves. Parts (c) and (d) are now well rehearsed by candidates on this paper and fully correct solutions were the norm. A few failed to comment on the change of sign in part (c) and there were some errors in accuracy in part (d). A very small number of weaker candidates in part (d) simply substituted 1, 2, 3 and 4 for x.

14. This part of the syllabus undoubtedly causes candidates much difficulty but it was disappointing to see so many unable to sketch $y = \ln 3x$ correctly, clearly showing it hit or pass through the y-axis, or having the wrong curvature. Finding the value of p was also too often incorrect. Full marks were extremely rare in part (b), although the first two marks were often gained. A common error was to produce an equation for the normal which was non-linear. Parts (c) and (d) were more familiar and answered much better, although errors such as $x^2 + \ln 3x = 0 \Rightarrow e^{x^2} + 3x = 0$ in (c) and the misreading of e^{-x^2} as e^{+x^2} or, more likely, incorrectly using a calculator to evaluate e^{-} in part (d) were quite common.

15. No Report available for this question.

48. No Report available for this question.